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ON TESTING LOCAL QFT AT LHC [1]

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Abstract

We discuss the importance of measuring $\rho = ReF/ImF$ at LHC to test the forward dispersion relations and local QFT. It is pointed out that at LHC we can reach a short distance domain that has not been pre-explored by QED. This is in contrast with all previous tests of the dispersion relations. We argue that the most likely property of QFT to fail is polynomial boundedness. In a theory with 'fundamental length', R, we study the consequences of having exponential behavior in the amplitude of the form $(\exp \pm i\sqrt{s}\, R)$, as suggested by different models. We show that such a behavior makes a significant and measurable contribution to ρ even at energies where $(\sqrt{s}\, R)$ is still small, O(0.1).

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For over three decades, the forward dispersion relations have represented one of the few general rigorous consequences of local quantum field theory. Starting from the axiomatic formulation of QFT, one can establish that the forward scattering amplitude, $F(s)$, has the following properties: a.) $F(s)$ is an analytic function of s with two cuts on the real axis. b.) It satisfies the property of crossing symmetry. c.) The optical theorem gives $ImF = k\sqrt{s}\sigma_{tot}$, where k is the center of mass momentum. d.) $F(s)$ is polynomially bounded for large $|s|$ in the cut plane, $|F(s)| < C|s|^N$. These four properties and the Froissart-Martin bound lead to the dispersion relations which enable us to calculate ReF from the total cross-section.[1)]

Starting in 1960, these relations have been repeatedly tested at practically every major new pp or $\bar{p}p$ accelerator or collider, covering a C.M. energy range of $\sqrt{s} = 7GeV$ to $\sqrt{s} = 550GeV$. As can be seen in figure 1, these tests always led to a measured $(ReF/ImF) \equiv \rho$, which agreed with the 'theoretical' dispersion relations fit for ρ. However, all these tests were never preceded by a high level of expectation that the results could have turned out to be different. The reason for that was the continuing improvement in the status of the experimental and theoretical knowledge of QED. For example when ρ was measured up to $\sqrt{s} = 7GeV$, it was already expected from the agreement between calculations and experiment in QED, e.g. the μ-meson magnetic moment, that there was no breakdown in QFT at distances of order $(10\ GeV)^{-1}$.

At present we have the following situation. If there is a 'fundamental length', R, then from QED and the results of the muon magnetic moment we have

$$\alpha(m_\mu^2 R^2) \leq 10^{-8}; \tag{1}$$

where the right hand side comes from the experimental error in the muon moment. This leads us to the following estimate

$$R^{-1} \geq O(100\ GeV). \tag{2}$$

With model dependent arguments one can accommodate a fundamental length R such that $R^{-1} \approx 300 - 500\ GeV$, but not much better. For our purposes here we can certainly make the following conservative statement. *Today, we have no experimental evidence that can rule out the existence of a fundamental length, R, such that* $(R^{-1}) > 1\ TeV$.[2)]

Furthermore, we have already reached the end of the line, as far as learning more from QED about R. Magnetic moment calculations to $O(\alpha^5)$ will not help since at that level hadronic and electroweak contributions become significant. Similarly, improving our experimental error on the muon magnetic moment, will not help us beyond $R^{-1} \approx O(1TeV)$, since again we will have to deal with contributions from non-electromagnetic processes that we cannot accurately calculate.

With the construction of the LHC, $\sqrt{s} \cong 15\ TeV$, we will be able to explore a short distance domain about which we have little previous knowledge. This is in sharp contrast to all the past tests of the dispersion relations and QFT, which dealt with length scales that

had been pre-explored by QED. In fact UA4/2 with $\sqrt{s} \cong 550\ GeV$ was more or less on the dividing line.

There are four other factors which make the measurement of ρ at LHC even more compelling. We list them here:

1.) At $\sqrt{s} \approx 550\ GeV$, $\rho_{\bar{p}p}$ is small[3], $\rho_{\bar{p}p} = 0.135$. In addition ρ decreases slowly (logarithmically). The phase of F, ϕ, with $\phi_{\bar{p}p} = tan^{-1}(1/\rho) \approx 83^o$, at $\sqrt{s} \approx 550\ GeV$. Also for $\sqrt{s} > 200\ GeV$ a good fit for ρ is $\rho \cong c[ln\ s]^{-1}$. These facts will turn out to be very useful below.

2.) Cosmic ray data on σ_{tot} indicate that $\sigma_{tot}(pp)$ continues to increase much beyond s_{LHC}. It grows like $(ln\ s)^{\alpha}$. While a range of α's are allowed, there is no doubt about the fact that the increase continues.[4] This limits the options we have in the calculated value of ρ at LHC energies.

3.) At LHC we for the first time reach the region of large $ln\ s$. Indeed we have

$$ln(s_{LHC}/m^2_{proton}) \approx 17,$$

and powers of $(ln\ s)$ differ significantly. This puts us in a domain where the phase relations[5] derived by Kinoshita and the author become useful tools.

4.) From the theoretical point of view, there are recent results which make the realm of superstring theories accessible to LHC. Of importance here is the possible existence of a new internal dimension at energies of a few TeV, i.e. low compared to the Planck mass. This new compact internal dimension could have a radius, R, with $R^{-1} \approx O(1\ TeV)$, and space time given by $M_4 \otimes S$. The new dimension is supposed to lead to supersymmetry breakdown. Such a suggestion, made earlier by several people, initially had several problems. However, it has recently been revived by Antoniadis[6]. He was able to present it in a form that not only overcomes most of the difficulties but also converts it into a viable proposal.

The first question we must answer is how will the dispersion relations fail. Which of the four properties of $F(s)$ listed in the first paragraph of this paper will fail? It is highly unlikely that $F(s)$ in any viable theory will have complex singularities on the physical sheet of the s-plane. Indeed such singularities would be a signal for unwanted physical states. It is also highly unlikely that crossing-symmetry fails. The third property of F, the optical theorem, is merely a statement on the conservation of probability and thus on very solid ground.

The most likely point of failure is the property of polynomial boundedness. This property is almost a direct consequence of the axiom on tempered distributions. It is the least physical of all the axioms. We can defend our choice of polynomial boundedness as the most likely pillar to fail by the following examples:

A. String Theory

The work of Gross and Mende[7] on scattering in string theory shows that polynomial boundedness fails in that case but at energies beyond the Planck scale. A similar situation was discussed much earlier when linearly rising Regge trajectories were first introduced.[8]

It should be noted that in string theory analyticity, crossing, and the optical theorem are not violated.

B. Non-Local Potential Scattering

In the Schrodinger equation we replace the local interaction term by a non-local one, i.e.,

$$V(|\vec{x}|)\psi(\vec{x}) \rightarrow \int d^3y V(|\vec{x}-\vec{y}|)\psi(\vec{y}). \tag{3}$$

The non-local potential is chosen to vanish outside a sphere of radius R,

$$V(|\vec{x}-\vec{y}|) \equiv 0, \ \ \text{for} \ \ |\vec{x}-\vec{y}| > R. \tag{4}$$

Then one can still prove that the scattering amplitude, $F(s)$, will still be analytic. However, polynomial boundedness is lost, and only $|F(s).exp(ikR)|$ is polynomially bounded.[9] Here $k \equiv \sqrt{s}$.

We can define F_{true} and F_{false} as:

$$F_t(s) \equiv F_f(s)e^{-ikR}. \tag{5}$$

$F_f(s)$ satisfies a dispersion relation, but $\mathrm{Im}F_f \neq k\sigma_{tot}$, and hence not guaranteed to be positive. However, for small k such that $(kR) << 1$, $F_t(s) \cong F_f(s)$, and if we only do our experiments at energies such that $\sqrt{s} << R^{-1}$, we will not detect any violation of the dispersion relations or locality.

C. Non-Local Field Theories

Almost all non-local QFT proposals which appeared before the introduction of the Wightman axioms had serious problems and violated many established principles. A noted exception is the more recent random lattice method of T.D. Lee[10] and collaborators, where a fundamental length is introduced without creating difficulties.

Non-local field theories are expected to have exponential behavior for $F(s)$ with a sign similar to that in the case of non-local potential scattering if the non-locality occurs in the time component.

D. New Compact Internal Dimension

In this case we have no violation of locality. However, polynomial boundedness could still breakdown due to the compact nature of the extra dimension. While the situation is not clear without a concrete theory that can be analytically studied, one can still make the following remark. Namely, the sign in the exponential will be the opposite to the one discussed above if polynomial boundedness breaks down due to the extra dimension. If we take seriously the short distance behavior of the amplitude in x-space as given by $\theta(x_o^2 - \vec{x}^2 - R^2\phi^2)$, where ϕ is the angle on S, as compared to $\theta(x_o^2 - \vec{x}^2)$ in the M_4 case, then we would have the situation where

$$F_t(s) = F_f(s)e^{i\frac{\sqrt{s}}{2}R} \tag{6}$$

This will give a different and distinct signal for ρ when $\sqrt{s}R$ is non-negligible.

In a recent paper[11)] we have studied non-relativistic potential scattering on $R_3 \otimes S$, and we get some significant changes in the properties of the forward scattering amplitude that are due to the extra compact dimension.

In the rest of this paper we shall assume that polynomial boundedness does indeed break-down at some energy R^{-1}. Taking the specific ansatz, inspired by our non-local potentials, we show numerically that even at energies such that $(\sqrt{s}R) \approx (1/10)$ there will be a strong and observable change in ρ.

We start with the ansatz:

$$F_t(s) \equiv F_f(s) e^{\mp i \frac{\sqrt{s}}{2} R}. \tag{7}$$

Here F_t is the 'true' amplitude, and $\mathrm{Im} F_t \equiv k\sqrt{s}\sigma_{tot} > 0$. On the other hand $F_f(s)$ is polynomially bounded, satisfies the dispersion relation, but $\mathrm{Im} F_f$ does not satisfy the positivity condition.

Equation(7) looks at first sight like a tautology. It is just a definition of $F_f(s)$. However, because of the special properties of ρ listed earlier, we can still learn some interesting things which are experimentally quite relevant. This will be shown immediately.

There are two cases to be calculated separately. They are distinguished by the sign of the exponential in equation (7).

Case I:

Here we have,

$$F_t(s) = F_f(s) e^{-i \frac{\sqrt{s}}{2} R}. \tag{8}$$

This is the case inspired by the example of non-local potentials. There are two energy regions to consider, low energies, $\sqrt{s}R << 1$, and a transitional region, $0.01 < \sqrt{s}R < 0.4 < 1$.

A. Low Energy Region:

Here $(\sqrt{s}R) << 1$, and hence $F_t \cong F_f$ and $\rho_t \cong \rho_f$.

B. Transitional Region:

This is defined as the energy range

$$0.01 < \sqrt{s}R \leq 0.4. \tag{9}$$

Essentially it is the region where $\sqrt{s}R$ is non-negligible but still below the threshold for producing excitations, $m_n^2 = n^2/R^2$. From eq. (8) we have,

$$ImF_f(s) = k\sqrt{s}\sigma_{tot}(s)[cos\frac{\sqrt{s}}{2}R + \rho_t(s) sin\frac{\sqrt{s}}{2}R], \tag{10}$$

which can be written as

$$ImF_f(s) = k\sqrt{s}\sigma_{tot}[1 + \rho_t(\frac{\sqrt{s}}{2}R) - \frac{1}{2}(\frac{\sqrt{s}}{2}R)^2 + O((\frac{\sqrt{s}}{2}R)^3)]. \tag{11}$$

It is now clear that as long as $(\sqrt{s}R/2) < 0.2$, and $\rho_t < 0.35$ (an assumption which we will show later is not needed), then in the transitional region we have

$$ImF_f = k\sqrt{s}\sigma_{tot}[1 - O(2\%)], \tag{12}$$

with $\sqrt{s}R < 0.4$.

Since $F_f(s)$ satisfies the dispersion relation, we can now extend the results into the transitional region where the optical theorem is still approximately good, and get

$$\rho_f(s) \cong \rho_{fit}(s), \quad \text{for} \quad 0.01 < \sqrt{s}R \leq 0.3. \tag{13}$$

Here ρ_{fit} is the value of ρ obtain from the standard dispersion relation fit shown in figure 1.

The true and false phases are related by

$$\phi_t(s) = \phi_f(s) - \frac{\sqrt{s}}{2}R. \tag{14}$$

But $\phi_f \equiv tan^{-1}(1/\rho_f)$, and in the transitional region using (13) we get

$$\phi_t(s) = tan^{-1}(1/\rho_{fit}) - \frac{\sqrt{s}}{2}R; \quad 0.01 < \sqrt{s}R < 0.3. \tag{15}$$

Using $\rho_t \equiv cot\phi_t$, we get ρ_t in the transitional region. The result is shown in figure 1, for the case where we choose $(R^{-1}) = 12\ TeV$. Even with that small length, by the time $\sqrt{s} = 4\ TeV$ the value of ρ_t is about 1/3, almost 2.5 times larger than UA4/2. The remarkable thing is that this occurs even when $\sqrt{s}R \leq 1/3$. In fact we begin to see a measurable effect on ρ when $\sqrt{s}R \approx 0.1$. Hence one gets a signal even when $\sqrt{s}$ is an order of magnitude below (R^{-1}).

Finally, we show that the assumption about ρ_t which we made in estimating the r.h.s. of eq. (11) and arriving at eq. (12) is not needed. All one need to do is to divide the interval $0.01R^{-1} \leq \sqrt{s} \leq 0.4R^{-1}$ into ten intervals and carry out the calculations from Eqs. (11)-(15) repeatedly starting from $\sqrt{s} = 0.01R^{-1}$. This way we can guarantee that the estimate (12) is correct throughout the transitional region.

Case II:

This is the situation that will obtain if polynomial boundedness is broken by an extra internal dimension. Here we have

$$F_t(s) = F_f(s)\ e^{+i\frac{\sqrt{s}}{2}R}. \tag{16}$$

Again $\rho_f \cong \rho_{fit}$ for $0.01 < \sqrt{s}R < 0.3$, but now ρ_t decreases and indeed becomes negative. The result is shown in fig.1. Again the signal is remarkable and the fact that ρ_t becomes negative allows us to make use of the inequalities of ref. 5.

In conclusion we state that the exponential behavior of $F(s)$ leads to a phase that is linearly dependent on $\sqrt{s}$, unlike the case where polynomial boundedness leads to a logarithmic behavior of $argF$. Regardless, of the result a measurement of ρ_{pp} at LHC energies will be quite important. Either $\rho_{expt} = \rho_{fit}$ at LHC, which effectively gives us a very high precision test of locality in QFT, $R^{-1} = O(10^2)\ TeV$ or $R = O(10^{-19})cms$. A result QED cannot match. Or, we get a disagreement between ρ_{expt} and the dispersion relation result which would mean a breakdown in QFT, and possibly suggestions of new structures in space-time at short distances.

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References

[1] The first proof of the dispersion relations is due to N.N. Bogoliubov, see N.N. Bogoliubov and D.V. Shirkov, *Introduction to the Theory of Quantized Fields*, (Interscience Publishers, London, 1959).

[2] T. Kinoshita, *Quantum Electrodynamics*, T.K. Editor, (World Scientific, 1990), pp. 471.

[3] Giorgio Matthiae, these proceedings (l993 edition), pp. 177-187.

[4] T.K. Gaisser, U.P. Sukhatme and G.B. Yodh, Phys. Rev. **D36**, 1350 (l987).

[5] N.N. Khuri and T. Kinoshita, Phys. Rev. **B140**, 706 (l965).

[6] I. Antoniadis, C. Munoz and M. Quiros, Nucl Phys. **B397**, 515 (l993).

[7] D.J. Gross and P. Mende, Nucl. Phys. **B303**, 407 (l988).

[8] N.N. Khuri, Phys. Rev. Lett. **18**, 1094 (l967).

[9] N.N. Khuri, Thesis (Princeton University, 1957), unpublished.

[10] R. Friedberg and T.D. Lee, Nucl. Phys. **D225**, 1 (l983).

[11] N.N. Khuri, "Potential Scattering on $R_3 \otimes S$", preprint in preparation.

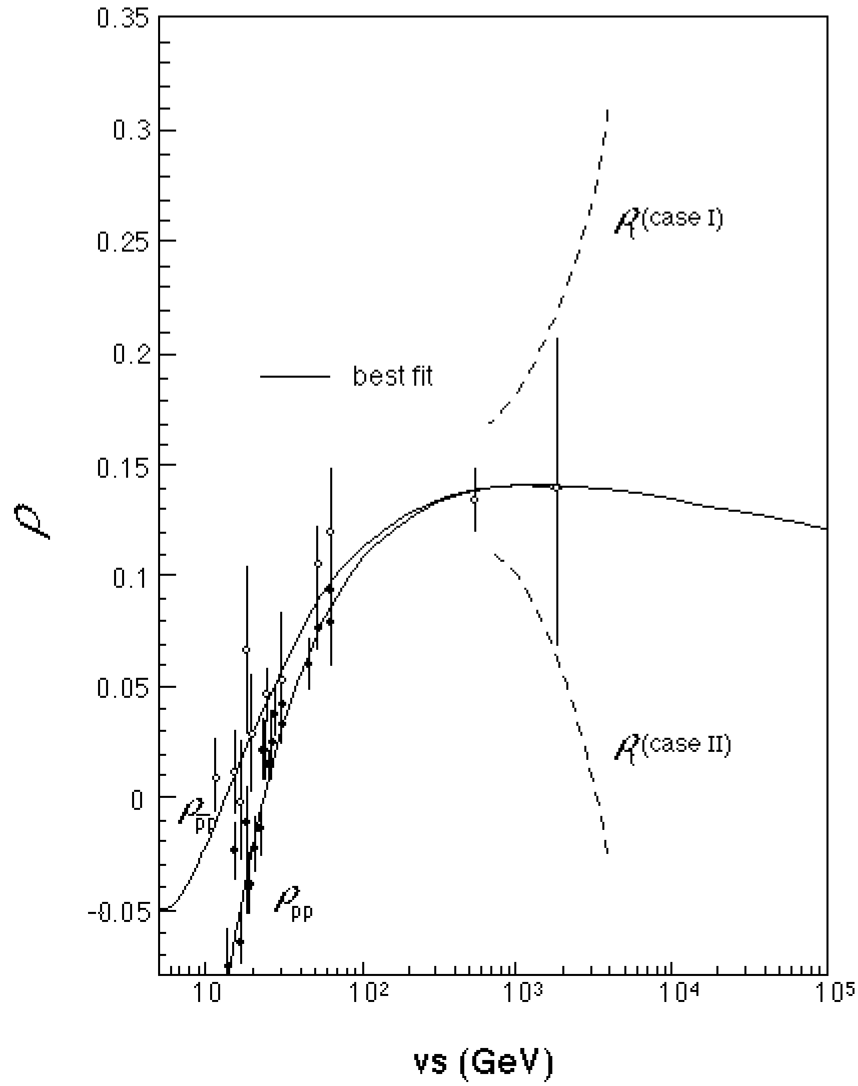

Figure 1: The curves give the best dispersion relation fit for $\rho_{\bar{p}p}$ and ρ_{pp}. The dashed lines represent our calculation of ρ_t for (R^{-1})=12 TeV, for both case I and II.

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